Samlingsförteckning: Radarupptäckt av artilleriraketer

Denna rapport behandlar en radarsensors förmåga att upptäcka 107 mm raketer beroende på hur sensorn positioneras i förhållande till skyddsobjektet. Fältförsök, underrättelser och stridserfarenheter har visat att dessa raketer är vanligt förekommande samt svåra att detektera med radarsensorer. En modell för hur räckviddsökning beror på olika sensorpositioner har skapats genom att använda dokument från USA och forna Sovjetunionen beskrivande ballistik tillsammans med teorier för hur räckvidd påverkas av radarmålarea (RCS) samt en beskrivning av RCS tillhandahållen av FOI. Resultat från körningar i MATLAB visar att sensorpositioner inom 300 meter från skyddsobjektet är fördelaktiga vid en skottvidd av 3000 meter. Som tumregel för att uppnå maximal sensorprestanda bör sträven vara att placera sensorn på ett avstånd från skyddsobjektet underligande 10% av förväntad skottvidd.

Nyckelord: C-RAM, raket, artilleri, radar, radarmålarea, upptäckt.
Radar detection of artillery rockets

Abstract:

This report examines how a radar sensor’s ability to detect 107 mm rockets depends on sensor positioning in relation to the protected asset. Field trials, intelligence and combat experience have shown that these rockets are commonly used and among the most difficult to detect with radar sensors. By using U.S. and U.S.S.R. documentation on rocket ballistics together with existing theories of detection range dependence on radar cross section (RCS) and a RCS description provided by FOI, a model for range gain for various sensor positions is constructed. MATLAB results shows that sensor positions within 300 meters of the protected object at a firing range of 3000 meters are favourable. As a rule of thumb, to achieve maximum sensor performance, the sensor should be placed not farther from the protected asset than 10% of expected rocket firing range.

Keywords: C-RAM, rocket artillery, radar, RCS, detection.
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Introduction

"O! say can you see by the dawn's early light, What so proudly we hailed at the twilight's last gleaming, Whose broad stripes and bright stars through the perilous fight, O'er the ramparts we watched, were so gallantly streaming? And the rockets' red glare, the bombs bursting in air, Gave proof through the night that our flag was still there; O! say does that star-spangled banner yet wave, O'er the land of the free and the home of the brave?" (Key, 1814)

The siege of Fort McHenry on September 13 and 14, 1814 was not the first time artillery rockets were used in war, but the poem they are mentioned in is well known. Even though the rockets only caused four deaths during this siege (Schombert), the fact that Key mentions them in his poem exemplifies their psychological effect.

Artillery rockets used before WWII where fairly small, fin-stabilized and carried an incendiary charge intended to ignite enemy ships and buildings. During WWII the rocket artillery was developed by many countries, with the Russian M-13 “Katyusha” being the most famous, lending it’s nickname to rocket artillery in general.

Early WWII rockets were fin-stabilized and launched from a rail, as is the case with the above-mentioned M-13. In order to improve accuracy, a slow spin was induced to the rockets as they developed and eventually spin-stabilized rockets without fins emerged. In order to make them aerodynamically stable, they needed to have a high spin rate. By using eccentrically and slanted mounted nozzles the rocket motor will give both forward thrust and angular momentum, which induces spin to rates of 300 rps$^1$ and more (Dullum, 2010).

Due to the ease of operating small spin-stabilized artillery rockets they have become widely spread and used in a number of conflicts. Conventional armies in many countries operate a wide variety of rocket launchers, from the smallest man-portable single-round systems to large towed or vehicle mounted with 40 launch tubes or more. There are no international conventions dealing with rocket artillery in specific and there is an excess of artillery rockets in the world, especially in areas that have seen a conflict during the past 30 years. This makes artillery rockets fairly easy to come by even for irregular forces,

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$^1$ Revolutions per second
criminals and terrorist networks (O’Malley, 1994). Afghanistan and Iraq are both areas where this is true.

In Afghanistan insurgents use artillery rockets to target civilians, ANSF\textsuperscript{2} and coalition forces. With the deployment to Afghanistan, this threat has become a reality to Swedish troops and a factor that has to be handled.

In regular warfare hostile artillery is normally handled by counter-battery fire, which means detecting the position of the opponents artillery and engaging it with own artillery. This approach is in some cases applicable in asymmetric warfare as well, but the nature of such conflicts complicates the use of own artillery. The difficulty to distinguish insurgents from civilians and the risk for fratricide are paramount factors, since misdirected use of offensive force may be counter effective to the strategic mission. Winning the hearts and minds of the population is not easy when you shell them with mortar. This is not a new problem, both USA and the former Soviet Union have experience from this approach and its drawbacks as described by (Bracco, 2008);

> Between the wars in Vietnam and Afghanistan, we have seen several methodologies which have been unsuccessful for fighting an insurgency. Two valuable lessons learned from the American experience in Vietnam and the Soviet experience in Afghanistan are that the indiscriminate application of firepower and firepower used against the population are roads which lead to failure. (Bracco, 2008)

One driving factor in C-RAM\textsuperscript{3} is the need to protect personnel. Counter-battery fire does not include warning and protection of those targeted by the incoming ordnance, but this is of large concern in the C-RAM concept. During the last decades, the demands for minimizing the risk for personnel have become more important. In general terms this means that more attention has been paid to protection in all forms than to firepower and manoeuvrability. This is the driving factor in developing both equipment and tactics. The ability to strike before the opponent is usually the best protection, but the nature of conflicts today may prevent this method. (Swedish Armed Forces, 2010)

Systems for protection need to be flexible and scalable to handle diverse threats in different types of conflicts. In conflicts where warfare is asymmetric,

\textsuperscript{2} Afghan National Security Forces

\textsuperscript{3} Counter-Rocket, Artillery and Mortar
protection from simple traditional and improvised weapons will be most important. Strict ROE\(^4\) will probably limit the possibilities for pre-emptive offensive actions, limiting us to act in self-defence, which in turn will enhance our exposure to the adversary’s fire. (Swedish Armed Forces, 2010)

Recent British operational experience (Joyce, 2012) indicates that a warning more than five seconds prior to impact is sufficient for personnel to assume a low position face-down which in turn will reduce casualties.

At present day the Swedish Armed Forces have two systems with RAM-detection capability fielded; Giraffe AMB and ARTHUR. Both systems have verified capability against RAM-targets. However, short-range rockets have proved to be difficult targets to detect due to low trajectories and short flight times (Humeur, 2011).

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\(^4\) Rules of Engagement
Scope
This report discusses how sensor performance varies with rocket attitude and motion with respect to RCS\(^5\) characteristics and how sensor tactics can be applied to achieve maximum sensor performance.

Sensor systems of interest are Swedish radar sensors with capability of detecting rockets, artillery and mortar. Existing systems are Giraffe AMB and ARTHUR. Results should be applicable to other sensors with the same RF\(^6\).

Fin-less, spin-stabilised 107 mm artillery rockets are of widespread use (O'Malley, 1994) and are experienced as very difficult targets for radar sensors. For these reasons this type of rocket is chosen as subject for this report. Results may be applicable to other spin-stabilised rockets, but this is not discussed further in the report.

Since early detection is critical for survival of personnel, the report will focus on rocket behaviour in the first part of the rocket trajectory only (Joyce, 2012) (Humeur, 2011).

Questions
To focus the work of this report two questions are formed:

- Is it possible to increase radar sensor detection performance versus 107 mm spin-stabilised rockets through optimised positioning of the sensor in relation to the protected asset?
- How can this be applied to field units?

\(^5\) Radar Cross Section

\(^6\) Radio Frequency
Purpose

In modern society, dramatic occurrences get intense media coverage and information technology make this available to a large amount of people in very short time. Social networks on the Internet and other available communication means makes interaction between people easier and faster than ever before. Examples can be seen that public opinion forms fast and has impact on politics, justice and military operations.

The affairs of Dominique Strauss-Kahn that eventually led to his resignation from the post as head of IMF\(^7\), as well as making a French presidential campaign impossible would probably never have had the same impact a hundred years ago. The February 2012 shooting of Trayvon Martin in Florida is another example of how public opinion can form quickly and influence the judicial system.

These aspects must be taken into consideration in military operations as well. Public opinion can quickly form and alter the course of events and decisions in a way negative to the objectives of a mission. It is equally important that personnel participating in these operations feel that actions are taken for their security, otherwise recruiting volunteers will be increasingly hard.

Since the end of WWII, the acceptance for casualties in war has drastically decreased in society. This is discussed by several modern military thinkers, like Merom and Münkler (Pekkari, 2009).

Fortunately Sweden has had comparably few casualties during the operation in Afghanistan, but those casualties have been given a lot of medial and public interest. Equally fortunate is the fact that very few indirect fire attacks have been performed against Swedish personnel and installations. There have, however, been some attacks on installations with Swedish troops like the one at Camp Marmal in May 2011 (Swedish Armed Forces, 2011). The author of this paper was stationed in northern Afghanistan during the final weeks of writing this report and during these weeks several IDF\(^8\) attacks were carried out using 107 mm rockets, one of them targeting Swedish troops in RCN\(^9\).

\(^7\) International Monetary Fund

\(^8\) Indirect fire

\(^9\) Regional Command North
Even though Swedish GBAD sensors have never been deployed in operations like this, the fact that the GBAD regiment has been given overall responsibility for C-RAM in Sweden implicates that efforts should be made to get a deeper understanding of factors influencing C-RAM capabilities.

The purpose of this paper is to gain more knowledge concerning radar detection of rockets in order to improve methods and tactics for sensor planning. This knowledge will, hopefully, contribute to save lives in areas where artillery rockets are a threat and radar sensors are used to detect them. In order to make the discussion and conclusions available to as many as possible of Swedish collaborators in the C-RAM community, the report is written in English.

The main audience for this report is believed to be C-RAM operators, planning officers with C-RAM responsibilities and other people with some experience from radar sensors and tactical use of such.
Existing research and theories

Historically, development of rocket artillery has been focused on production of rocket artillery systems. A complex pattern of laws of physics have influence on rocket ballistics, most prominent are flow dynamics together with mechanics. Research on spin-stabilised rockets is a specific branch of applied physics relying on accepted definitions and formulas well established within the scientific community.

During recent years, part of this focus has been shifted to the protection from them.

Recent Swedish military development has in many aspects been driven by needs of ISAF\textsuperscript{10} forces. With the deployment to Afghanistan, Swedish personnel and equipment has been put to tests in harsher situations than for a long period of time. The consequences of environmental conditions and asymmetric warfare have become evident to those serving in Afghanistan and eventually these experiences have become widely spread in the organisation. It is believed that this is not a temporary condition, but the way development of Swedish military abilities will look like (Swedish Armed Forces, 2010).

Enhanced tactics, new procedures and upgraded equipment have become part of everyday life in most Swedish units. Amongst threats needed to be handled is indirect fire. C-RAM has been put forward as an important capability, especially in operations like those in Afghanistan. Many countries put a lot of effort into the field of handling incoming ordnance; Germany has invested more than €100 million in development and procurement of the MANTIS system, USA has allocated vast sums for procurement and fielding of the Centurion system and Israel is putting a great deal of prestige into the development of its Iron Dome project. Sweden has been developing C-RAM capability since 2008, starting with a study that was finished and reported in 2009 (Swedish GBAD regiment, 2009).

It is also interesting to note the U.S. fundings for upgrade of the AN/TPQ 36 Short range Firefinder radar the recent and coming budget years: 2011 – 286 mn, 2012 – 338 mn, 2013 – 299 mn, 2015 – 348 mn, 2016 – 124 mn, 2017 – 153 mn. In total, this means a spending of more than 1.5 billion USD on just one C-RAM sensor system. Equally interesting is that this is done with focus on operation in high-clutter environments and detection of low-quadrant

\textsuperscript{10} International Security Assistance Force
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Elevation targets (US Army, 2012). These two factors are paramount to increasing the capabilities for rocket detection.

Apparently, handling of threats posed by improvised indirect fire is of great concern to many countries involved in operations that can be described as asymmetric. To facilitate development, the Trilateral Radar Data Interchange MoU\(^{11}\) concerning interchange of data and experience between USA, UK and Sweden was signed in October 2009 (Trilateral Radar Data Interchange MoU, 2009).

**C-RAM**

DAMA\(^{12}\) is one out of ten NATO working groups under the DAT\(^{13}\) programme. It was initiated by the Netherlands due to the increasing number of terrorist mortar attacks in Iraq and Afghanistan and is presently led by Norway. The work within DAMA is focused on how to apply new and future technology to position indirect fire POO\(^{14}\) and POI\(^{15}\) and to react against the attacker, the projectile or to protect own personnel. Within DAMA the C-RAM concept is based on seven pillars;

- Prevent
- Detect
- Warn
- Intercept
- Protect
- Attack
- C\(^2\)

\(^{11}\) Memorandum of Understanding

\(^{12}\) Defence Against Mortar Attacks

\(^{13}\) Defence Against Terrorism

\(^{14}\) Point of Origin

\(^{15}\) Point of Impact
Preventing deals with measures taken in order to deny an adversary the opportunity to use RAM weapons against protected assets. Possible ways to achieve this are large perimeters around camps, cooperation with the local inhabitants and collection/disposal of ordnance. Detection of C-RAM can be performed in various ways; by acoustic means, by radar sensors and by EO\textsuperscript{16} systems, just to mention a few. The aim is to determine the presence of a threat and to position it in time and space. The purpose of warning is to alert those posed to threat as fast and accurate as possible. False alerts will have negative effects, as will too late or missing warnings.

To intercept a threat in mid-air it must have been detected with enough accuracy first. Deflagration of the incoming ordnance is the most desirable effect, since it will decrease the risk of fratricide compared to a detonation. Protection can be obtained by fortifications, but training is another action that can be considered as a protective measure. A warning needs to be followed by correct actions taken by the personnel in order to be effective.

Attacking the POO is basically what is usually known as counter-battery fire. In historic conventional wars this has been the main course of action, but against an adversary firing rockets initiated by time-delay devices from densely populated areas it will probably cause more problems than it solves (Bracco, 2008).

Command and control is more or less integrated in all the other six pillars and deals with issues like transmission systems for warning, computer aided tools for determining optimal sensor positioning and so forth.

The DAMA definition of C-RAM will be used within this report.

The scope of this report puts it mainly within the “Detect” portion of C-RAM as defined by DAMA, even though $C_2$ may be considered partly applicable as well.

There are studies made concerning C-RAM capability in Sweden, a summary of these was made by (Pekkari, 2009) and in his report he points out the need to further investigate the subject.
**Rocket artillery**

Experiments with rocketry are believed to be closely linked to the development of gunpowder (Sinyarev & Feodosyev, 1966). The invention of both is believed to be Chinese and documents describing the use of military rockets in Europe as early as the 10th century exists. Rocket artillery development was put aside for several centuries due to the invention of smoothbore artillery, and it was not until the end of the 18th century that European interest in rocket artillery was evoked. In their wars in India, the British Army faced the Indian lightweight artillery rockets, inspiring LtCol William Congreve to start improving rocket design for use in the British Army.

Development of rocket artillery was at its peak during WWII and the following decades. Many scientific reports from the 1950’s and 1960’s can be found, mainly Soviet and US documents. Basically, they are similar both in theories and conclusions, and being independent sources they verify each other. It should be noted that the Soviet documents used are machine translations provided by the US Air Force, but the translations are made after the US reports concerning these matters.

These studies where mainly intended for the purpose of constructing rocket artillery systems, for example M270 MLRS\(^\text{17}\), but some ideas and equations should be applicable even to a rocket launched with improvised means, for example a car jack or a sandbag. The aerodynamics and ballistics of the rocket are interesting since they will influence the RCS due to the fact that the rocket has a nutational movement in flight. Documents from Aberdeen Proving Ground (US) and the work of A.A. Dmitriyevskiy, V. I. Sinyarev and G.B. Feodosyev (USSR) were used to describe rocket ballistics. Most of these references are from 1960-1980, since this period seems to be when most of the modern research in rocketry was performed and documented.

**RCS**

Theories concerning RCS measurement and estimation have evolved in close relation to computer development. Precise methods for RCS estimation are very complex operations involving computer intensive calculations. More powerful computers mean more exact calculations in shorter time. Browsing through libraries for scientific reports in the field of computer calculations of RCS yields a vast amount of information. Performing calculations of RCS is

\(^{17}\) Multiple Launch Rocket System
not within the scope of this report, why it has been judged sufficient to understand the complexity of the matter and rely on information provided by FOI.

RCS basics are comprehensively described by (Knott, 1990) and the influence of target fluctuation as defined by (Swerling, 1954) is generally accepted amongst radar scientists.

**Laborations**

RCS measurements of 107 mm rockets have been performed at FOI during 2010 for frequencies applicable to this report. The exact figures are classified, but FOI has provided what they describe as an “accurate enough estimation verified by measurements” (Gadd, 2012). These measurements are one of the pillars to this report and the validity of the results is vital since much of the discussion rely on them. FOI is one of Europe’s leading research agencies for defence studies and have a long experience with RCS measurements. In several reports, for example (Löthegård, 2000), eventual shortcomings concerning equipment and methods are discussed. The number of reports concerning reliability of results from FOI in Linköping indicates a commitment for accuracy and results from the installation have been used successfully for far more advanced objects than 107 mm rockets with good results.

Field trials with Giraffe AMB versus 107 mm rockets have been performed several times at Salisbury Plain, UK, during the Brilliant Sky exercises. It would have been interesting to analyse the data from these trials, but that is not possible since that data is classified. However, since the author of this report was responsible for Swedish evaluation during Brilliant Sky 5 in March 2011, experience from live trials are part of the foundation for this paper and the main reason that the need for this report became evident (Humeur, 2011). The fact that the author has been deeply involved in Swedish development and testing of Giraffe AMB C-RAM capability may have influenced this report. Aware of this risk, care has been taken to avoid tendencies.

**Combat experience**

Within the Trilateral Radar Data Interchange MoU, lessons learned from US and UK forces have been transferred to Sweden. This information is not possible to reference in this document due to the fact that it is classified. As is

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18 Swedish Defence Research Agency
the case with trial results, discussions concerning lessons learned have sparked ideas within this report.

Informants concerning unclassified combat experience are still in service with their respective units. This probably means that they are under some sort of agreement not to share classified information, which in turn most certainly will have impact on their statements. It is also important to notice that the author has tried to exercise carefulness according to how questions where formulated and what answers where possible to use in this unclassified report. Naturally, this introduces even more reasons to be critical to this type of information. On the other hand, the sources have done active service in relevant conditions and have experience that only a few in the world possess. This makes their contribution very valuable and as stated earlier, without their experience this report had never been written.
Method

Military technology is in many aspects a combination of theory, tactics and practice. This of course influences the structure of this paper.

The Swedish GBAD\(^{19}\) Regiment is responsible for C-RAM in Sweden and has built up experience in this field, mainly by participating in exercises. Results from UK live fire exercises with 107 mm rockets fired from improvised launchers have been paramount to this report (Humeur, 2011). Due to classification reasons it is not possible to reference these results, but experience from them was used in as many aspects as possible.

Lessons learned from existing operations were used to some extent, but the problems with these are discussed further in “Combat Experience”.

Initially, a thorough study of existing theories concerning rocketry and RCS was made. The main purpose of these studies was to relate phenomenon experienced during live fire trials to existing research and models and to build a foundation for further discussion.

Results from measurements performed by FOI concerning RCS are used to describe the rocket from the radars electromagnetic point of view.

Information about rocket ballistics, rocket electromagnetic properties and general theories about RCS and radar range is used to describe rocket characteristics and how it influences rocket detectability in terms of how it reflects electromagnetic emissions from the radar. A model is created and equations for resolving radar detection range gain are formulated. This model is evaluated in MATLAB for different values of the relevant parameters and representable plots of range gain are created. From this information, conclusions about radar positioning relative the protected asset are drawn.

\(^{19}\) Ground Based Air Defence
**Model**

Since, presently, the most probable scenario for deployed C-RAM sensor systems is camp protection for expeditionary forces, a model for a typical camp deployment is defined.

![Diagram of camp scenario](image)

**Fig. 1 Scenario**

Depicted in the centre of figure 1 is the protected asset, in this case a field hospital. The rocket system can be positioned anywhere along the circle, which symbolises shortest practical firing distance. Positioning of the sensor can be done anywhere within this circle and a line from the asset through the sensor will define \(0^\circ\) firing position angle.

MATLAB plots of aspect angle versus firing angle are then used to analyse how the sensor will perceive RCS for different distances from the protected asset. From this analysis, conclusions are drawn concerning favourable and less favourable positions for the sensor in relation to protected asset and rocket firing position.

**Restrictions**

Only radar frequencies in the mid C-band will be analysed, since those are applicable to the sensors discussed.
Available RCS data covers aspect angles 0°-90°, which means that only approaching rockets will be analysable. For this reason, only sensor-asset distances shorter than rocket-asset distances will be analysed. RCS calculations are not within the scope of this report.

Most probable firing range is set to 3000 meters. This value is used throughout the report. Implications of other distances, however, are discussed.

Since early detection is critical, only aspect angles immediately after launch are analysed (Joyce, 2012) (Humeur, 2011). For the short firing ranges that are applicable to this report, elevation at launch is 10 degrees or less (table 1). The elevation will influence perceived RCS, but the influence will be the same for all aspect angles, why elevation will not be considered in this report.

Smoke plume influence on RCS is not covered in this paper, even though metallic particles in the smoke may have an impact on perceived RCS. Signal processing in the covered radar systems is, from the author’s knowledge of the systems, focused on signature from the rocket itself. Due to this, the smoke plume is believed to be of less interest in this report since it would not be classified as a rocket.
Analysis

Radar generalised

In order to be able to track a target the radar needs some inputs. For the two systems discussed in this report, main input come from the reflected energy from the radar pulse that has hit the target. First of all, the received reflection must have sufficient amplitude. This amplitude varies depending on many factors such as distance, air moisture, RF and more. One important factor is RCS, which in turn is depending on the physical shape of the target, the aspect angle and RF. Most factors concerning received amplitude are hard to affect, but the aspect angle can, to some extent, be controlled by the way the sensor is positioned in relation to where targets are probable to appear and how they are believed to behave in air.

Giraffe AMB

The Giraffe AMB has been in service with the Swedish Armed Forces for almost a decade. It is a ground-based, mobile 3D radar sensor on C-band with capability to detect RAM-targets while simultaneously performing air-surveillance in a 360° sector. The system is equipped with audio systems for RAM alert in up to 10 different areas defined by the RAM operator. In order to provide an accurate prediction of POI it continues to track all detected RAM targets until impact, thus improving POO and POI precision during the RAM targets complete trajectory.

ARTHUR

The ARTHUR system has been in service for a few years more than the Giraffe AMB. It is in many aspects similar to the AN/TPQ 36 Firefinder used by the USA. The system scans the horizon in a 90° sector until it detects a RAM target. The target is then tracked until POO and POI can be calculated. Radar lobe elevation is achieved by change of RF, which makes the system vulnerable to limitations in available RF channels.

Radar range equation

The two-way radar equation can be written as

$$R^4 = \frac{P_t \times t_p \times G_t \times G_r \times \lambda^2 \times \sigma}{(4\pi)^3 \times k \times T_x \times D \times L_t \times L_\alpha}$$

where R is the maximum theoretical detection range in meters and $\sigma$ is the equivalent RCS in square meters. Since all parameters except $\sigma$ in the
right-hand part of the equation can be considered as constants in this report, the equation can be re-written as:

\[ R^4 = k_{RCS} \ast \sigma \]

\[ R = \sqrt[4]{k_{RCS} \ast \sqrt[4]{\sigma}} \]

\[ \Delta R = \sqrt[4]{\Delta \sigma} \]

concluding that the maximum detectable range changes in proportion to the fourth root of the RCS change. This equation will be used later in this report to convert RCS differences to relative range gain.

**107 mm spin-stabilised rockets**

**Ballistic properties**

Spin-stabilised rockets are designed to be launched from a canister, normally a tube with the same inner diameter as the rocket. The rocket motor produces forward thrust to propel it out of the tube and a small portion of the thrust is used to produce rotational movement. This is achieved through several off-centred, slanting motor nozzles. The rotational movement is used to stabilise the rocket, much the same way a rifled muzzle will spin-stabilise howitzer ordnance.
Length to diameter ratio should ideally be seven or less for stability reasons (Dullum, 2010) (Farrar & Leeming, 1983). Ideal theoretical length of a 107 mm rocket is due to this 749 mm. However, for reasons of maximum range and maximum payload, most 107 mm rockets found have lengths exceeding this. One example is TRB-107 manufactured by Roketsan, Turkey (Turkish Undersecretariat for Defence Industries), which has an overall length of 840 mm.

Rocket firing distance is determined by the elevation at launch. This elevation is normally achieved by elevating the launch tube, but under improvised conditions a sand pit or a car jack can be used. Shortest practical firing distance is 2000 meters, otherwise the low trajectory may cause the rocket to hit buildings or geographical obstacles before it reaches the target.

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<th>Launch elevation (°)</th>
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<td>35.0</td>
<td>36.2</td>
<td>1541</td>
<td>224</td>
<td>44.0</td>
</tr>
<tr>
<td>8.5</td>
<td>46.0</td>
<td>45.9</td>
<td>2500</td>
<td>233</td>
<td>56.2</td>
</tr>
</tbody>
</table>

Tab. 1 107 mm rocket suggestive firing table (Dullum, 2010)

Static and dynamic unbalance of the rocket may produce a nutational movement, which is most prominent immediately after launch and dampens out as rotation and velocity increases. The static unbalance basically depends
on whether the rocket's centre of gravity coincides with its geometric centre. Dynamic unbalance relates to linear and angular thrust misalignments.

The amplitude of this nutational movement depends on the mentioned parameters, of which few are known with certainty. Frequency of the nutational movement is depending on the rocket's angular velocity and dampening of this whirling motion increases with angular velocity, assuming that the rocket is aerodynamically stable. No detailed information on rotation rate for 107 mm rockets have been found and for this reason an interpolation of values for other calibre rockets described by (Dullum, 2010) has been made. Approximate angular velocity, \( \omega \), for a 107 mm rocket is estimated to 2500 rad/s based on an estimation that the rocket rotates with 400 rps.

The rocket's aerodynamic behaviour will influence its radar detectability, as discussed further later in this report.

**Radar properties**

It is assumed that the rocket's exterior is homogenous and conductive.

Reflection of electromagnetic energy from an object mainly consists of two parts, specular return and creeping wave. The specular return can be described by optical formulas but the creeping wave is frequency dependent where wavelengths close to the size of the target induces the strongest creeping wave (Avionics Department AIR-4.5, 1999).

For a perfectly conducting sphere, creeping wave and specular reflection go in and out of phase depending on the size of the object and its relation to the wavelength of the radar signal. For \( 0 > \frac{2\pi r}{\lambda} \gg 1 \), the Rayleigh region, RCS increases drastically to a peak near \( \frac{2\pi r}{\lambda} = 1 \). In the resonance region where \( 1 \gg \frac{2\pi r}{\lambda} > 10 \), RCS varies with the interference between the creeping wave and the specular reflection (Knott, 1990). For objects that fall within these regions calculations will be too imprecise to estimate RCS if the RF of the transmitter is variable, as is the case with the studied sensors.

To produce reliable estimations of target RCS it should ideally fall in the region where \( \frac{2\pi r}{\lambda} \gg 10 \). In this region the specular return is the dominating component to RCS and is described as the optical region.
The ARTHUR has a RF range of 5400-5900 MHz (SAAB Group) as well as the Giraffe AMB (SAAB Group), equalling wavelengths of 0.0508 m to 0.0555 m. For

\[
\begin{align*}
\lambda &= 0.053 \, m \\
2r &= 0.107 \, m \\
\frac{2\pi r}{\lambda} &\approx 6.3
\end{align*}
\]

which indicates that part of the RCS function of the 107 mm rocket for the studied RF falls in the resonance region (Knott, 1990). However, since the rocket is longer than 10 \( \lambda \) it should behave mainly as an optical scatterer. This leads to the conclusion that RCS estimations based on calculations for the optical region can be used to describe the 107 mm rocket for the relevant frequencies.

Equivalent RCS in the beginning of the boost phase will vary depending on aspect angle, \( \beta \).

Measurements and simulations performed by FOI in 2010 have been fused in order to produce figure 5 below.
Robert Humeur

Radar detection of artillery rockets

Noticeable in this graph is the fact that aspect angles between 10° and 70° are least favourable and therefore these are used as reference (range factor 1).

The graph is constructed by data for every 10 degrees, which means that data for angles in between will have to be interpolated. For this purpose two matrixes are defined; \textit{angle}, which is a 1:10 matrix containing the respective angles

\[ \text{angle} = [0 \ 10 \ 20 \ 30 \ 40 \ 50 \ 60 \ 70 \ 80 \ 90] \]

and \textit{rcs}, which is a 1:10-matrix with RCS-data converted to range factor where -30 dBsm is reference. We remember that

\[ \Delta R = \sqrt[4]{\Delta \sigma} \]

which yields that

\[ \text{rcs} = [\sqrt[10]{1} \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ \sqrt[10]{1} \ \sqrt[10]{100}] \]

With these matrixes, RCS data for all angles between 0° and 90° can be interpolated in MATLAB. Having tried different interpolation methods, “cubic” is judged to provide the most representative values and is therefore used in the calculations later in this report.

\textbf{RCS fluctuation due to nutational movement}

Since RCS varies with aspect angle, any movement of the target will cause a change in perceived RCS. The nutational movement of the rocket causes the RCS to fluctuate in a way that is determined by

- Stability factor, mechanic and dynamic
- Angular velocity

This fluctuation causes a fluctuation loss, as described by (Swerling, 1954) and the relatively high angular velocity, 2500 rad/s, implies that the rocket can be
considered as a Swerling case II. This means that RCS will fluctuate on a pulse-to-pulse basis, reducing gain from coherent signal integration and increasing gain from non-coherent integration. The high rate of rotation implies that diversity gain due to time diversity can be applicable where the numbers of samples available, $n_e$, will be (Barton, Cook, & Hamilton, 1991)

$$n_e = 1 + t_0/t_c$$

where $t_0$ is the available sample time, i.e. the time the antenna beam illuminates the target, and

$$t_c = \frac{\lambda}{2\omega a L_x}$$

where $\lambda$ is radar wavelength, $\omega a$ is the rate of rotation of the target and $L_x$ is the target dimension normal to the line of sight. Earlier, $\omega a$ was estimated to 2500 rad/s. For the Giraffe AMB with a 2,2° antenna beam and a maximum antenna spin rate of 1 rps, $t_0$ will be approximately 6 ms which results in more than 60 available samples per scan for 0° aspect angle and more than 400 samples for 90° aspect angle, which should be sufficient for good integration gain. However, this will not be investigated further in this report. It would be interesting to analyse if the difference in available samples for integration gives a difference in detectability, but this is judged to be beyond the scope of this paper.
Radar aspect angle

Rewriting the scenario from figure 1 in mathematical terms gives figure 6. The protected asset is defined as origo.

![Diagram of radar aspect angle](image)

**Fig. 6 Geometry and variables**

Rocket firing position is defined by the complete length of the trajectory:

\[ d \text{ (meters)} \]

and the angle relative x-axis

\[ \alpha \text{ (degrees)} \]

where

\[ 0° < \alpha < 360° \]

The sensor is positioned a distance

\[ a \text{ (meters)} \]

from the sensor position along the x-axis where

\[ 0 < a < d \]

The sensor-rocket aspect angle is defined as

\[ \beta \text{ (degrees)} \]
where

\[ 0^\circ < \beta < 90^\circ \]

Fig. 7 Trigonometric dependencies

Law of sine gives:

\[
\frac{a}{\sin \beta} = \frac{d_{\text{sens}}}{\sin \alpha} \]

\[
\sin \beta = \frac{a \sin \alpha}{d_{\text{sens}}} \]

Pythagoras law gives:

\[
d_{\text{sens}} = \sqrt{x_{\text{sens}}^2 + y_{\text{sens}}^2} \]

where

\[
x_{\text{sens}} = d \cos \alpha - a \]

\[
y_{\text{sens}} = d \sin \alpha \]

thus

\[
\beta = \arcsin \left( \frac{a \sin \alpha}{\sqrt{(d \cos \alpha - a)^2 + (d \sin \alpha)^2}} \right) \]
For different values of sensor distance, $a$, and rocket firing distance, $d$, it is now possible to calculate $\beta$. By interpolating range gain for every $\beta$ from the matrixes $rcs$ and $angle$, plots of range gain versus rocket firing angle can be made in MATLAB.

**Result and analysis**

Calculations in MATLAB have been made for various combinations of values $a$ and $d$ in order to find the relation between them and how they influence theoretical range gain. As stated earlier, a rocket firing distance of 3000 meters has been used. Plots in the following section have been chosen to represent values where interesting conclusions can be drawn and boundary effects can be identified.

For $d = 3000$ meters the resulting range gain factor for three different sensor-asset distances, $a$, is as plotted in figure 8.

![Figure 8 Range gain vs. firing position angle (1)](image)

From this plot it is noticeable that gain resulting from the high RCS when approaching aspect angles orthogonal to rocket trajectory are hard to achieve. Even if the gain exceeds 3.5 times in those cases, it is equally important to notice that for the rest of the rocket firing positions it is less favourable with a position far from the protected asset.

For positions closer to the protected asset, gain from aspect angles close to $0^\circ$ are prominent. From figure 5 we remember that RCS for $0^\circ$ was 10dB higher than the reference level, yielding a range gain factor of $\sqrt{10} \approx 1.8$, which is consistent with the maximum gain plotted above for the shorter distances.
Having examined the boundary values for $\alpha$, a look at intermediate sensor-asset distances shows that there will be limited possibilities to achieve favourable aspect angles.

![Graph](image)

**Fig. 9** Range factor vs. firing position angle (2)

The plot in figure 9 depicts how gain from aspect angles with high RCS is impossible to achieve for sensor positions 1000-2000 meters from the protected asset. In order to benefit from favourable aspect angles, sensor-asset distances of more than 2900 meters are needed, and even for those ranges the effect is only present for very specific angles. Looking at the plot in figure 10 and remembering the typical RCS distribution for a 107 mm rocket we realise this.

![Graph](image)

**Fig. 10** Aspect angle vs. firing position angle
As discussed earlier, gain from the higher RCS for aspect angles (70° to 90°) is only present for very long sensor-asset distances and only for very specific angles of $\alpha$. Unfortunately, rockets fired from these positions will be closer to the sensor than the radars shortest measurable distance, and therefore not detectable at all. For this scenario it is clear that gain from aspect angles of 70° to 90° is unachievable.

Having stated this, it is important to evaluate sensor positions close to the expected POI.

![Graph showing range gain vs. firing position angle]

**Fig. 11 Range gain vs. firing position angle (3)**

From figure 11 we can draw the conclusion that the closer to the protected object the sensor can be positioned, the better.

As stated previously, all plots are performed with rocket firing positions 3000 meters from origo. A change in this value will yield a proportional change in sensor-asset distance to achieve the same range gain, i.e. a firing distance of 2000 meters and a sensor-asset distance of 200 meters will result in the same range gain as 3000 meters and 300 meters for the same angles.

Therefore, as a rule of thumb, sensor-asset distances 10% or less of expected rocket firing distance should be strived for in order to benefit from range gain from favourable aspect angles. In figure 9 it can be seen that sensor-asset distances of 300 meters results in a minimum range gain of 1.2. Lower levels of gain are considered too small to have any practical effect on detectability in reality.
Normally, the firing distance is not known, why the shortest practical firing range should be used. This means that in order to optimize sensor positioning for early detection of 107 mm rockets, for which 2000 meters is believed to be shortest practical firing range (Dullum, 2010), the optimal sensor position is within 200 meters of the centre of the protected asset. Similarly, if a sensor is positioned at a defined spot, we know that attacks against objects within 200 meters of it will be detected with a higher probability than those against objects further away from the sensor for most cases.

It should be noted that the range gain factor is only a theoretical value used to compare the different sensor positions in relation to each other. The value should not be used for estimating real detection range since other effects like terrain, pulse length, peak power etc. need to be considered for this.

**Answers to research questions**

Two questions were formulated in the Scope section of this report. The first was:

- Is it possible to increase radar sensor detection performance versus 107 mm spin-stabilised rockets through optimised positioning of the sensor in relation to the protected asset?

  Yes, it is. For the defined scenario, applicable to protection of a camp or similar, the choice of a radar sensor position in the direct vicinity of the protected asset may increase detectability. Positioning the sensor closer to the asset than 10% of expected rocket firing distance will be beneficial.

The second question, relying on the above answer, was:

- How can this be applied to field units?

  By implementing this knowledge in existing Swedish GBAD method directives, it can be used as a guideline for sensor positioning during operations and training.
Conclusions

This report has shown that for a specific scenario it is possible to position a sensor system in a way that beneficial effects from target RCS can be achieved. The results are easily applicable to field units during operations and training, even though physical limitations (obstacles, infrastructure etc.) may impede optimal sensor positioning. The results from the report can also be used to determine in which areas assets will be best protected, given that sensor position is fixed and assets are positioned afterwards.

The use of a limited scenario has some advantages in the way it simplifies the analyse, but also makes the results less useful. In order to make the results applicable to a more diverse scenario it should be interesting to evaluate aspect angles of 0° to 180° for a wider set of RCS data for several different artillery rockets.

This report only uses RCS as reference for detectability. Other factors are not considered and weighted against each other, which is a weakness in the results. However, as discussed earlier, since aspect angle is one of the few parameters that the C-RAM planner can influence, the conclusions should be applicable for practical use.

Further analysis of how time diversity gain is applicable could provide more details and a more complete description of how detectability varies with aspect angle.

In this report, detection at launch is discussed. It would be interesting to simulate how detectability changes along the rocket trajectory for different sensor positions. In order to do this, more data on different artillery rockets ballistics (impulse, moment of inertia, nozzle cant angle etc.) needs to be analysed and a more complex model of detectability has to be constructed.

The initial purpose of this report was to analyse classified test data from live firing trials. For reasons of information security it was not possible to perform the analyse in the way intended. This still needs to be done and the work with this report has provided useful background information that will benefit that analyse.

It is recommended that the results from this report are examined, criticised and discussed by personnel in field units, Swedish and foreign.
References


Additional literature


Appendix I: MATLAB code

function [ beta ] = aspect( a,d )

%ASPECT Calculates sensor-rocket aspect angle, a is distance from sensor to protected asset in meters, d is rocket firing distance in meters. Returns beta in degrees.

% alpha is rocket firing direction in radians
% beta is sensor-rocket aspect angle in degrees
alpha=linspace(0,2*pi,360);
% creates angles 0-359 with one degree increment (in radians)
% xsens is the rocket firing position (x-axis) in relation to sensor
% xsens=d*cos(alpha)-a;
% ysens is the rocket firing position (y-axis) in relation to sensor
% ysens=d*sin(alpha);
% dsens is the distance from sensor to rocket firing position
% dsens=sqrt(xsens.^2+ysens.^2);
% beta is the angle between sensor beam and rocket trajectory
% beta=abs(90*a*sin(alpha)./dsens);
% With all expressions substituted, equation for beta reads;

beta=abs(asin(a*sin(alpha))./(sqrt((d*cos(alpha)-a).^2+(d*sin(alpha)).^2))))*360/(2*pi);
end
```matlab
%RANGEFACTOR_PLOT Produces a plot of range compared to "worst case"

% Loads the RCS model
load('\netapp01\homedir$\Stud\HSU12066\My Documents\MATLAB\107 mm rocket simulation\angle_0_90.mat')
load('\netapp01\homedir$\Stud\HSU12066\My Documents\MATLAB\107 mm rocket simulation\rcs_107_model.mat');

% Requests position data
a1=input('Distance from sensor to protected asset in meters? ');
d1=input('Rocket firing distance in meters? ');

% RCS range factor for aspect angles from ASPECT are interpolated from
% the RCS model
rcs_range=interp1(angle,rcs,aspect(a1,d1),'cubic');
plot(rcs_range);
```